On the impact of tax uncertainty on investment into carbon abatement technologies

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Introduction

- Carbon taxes are a key policy tool for fighting climate change (e.g. Nordhaus [1993], Golosov et al. [2014])
- Most of this work is concerned with optimal tax schemes for an efficient emission reduction (Nordhaus [1993], Golosov et al. [2014])
- In reality (environmental-) tax policy is a political decision that is affected by many factors such as political sentiment, outcome of elections, lobbying or international climate policy, so that future tax rates are stochastic
- In fact Climate Policy Uncertainty and its impact on asset prices and investor decisions has become an active research topic

Our contribution

- We study how uncertainty about carbon tax rates affects investment strategy of a electricity producer who can invest in abatement technology
- Investments are irreversible and subject to transaction cost $\Rightarrow$ Producer is faced with a *dynamic control problem.*
- Two approaches for *tax uncertainty*:
  i) taxes as a stochastic process with fixed dynamics, namely a finite state Markov chain (risk)
  ii) taxes as result of a differential game between producer and "nature" (uncertainty)
- Mathematical contribution. Analysis of the control problem and the differential game
- Financial contribution . Numerical experiments on the impact of tax uncertainty and of the structure of production and abatement technology on investment and emissions.

Related work

- Fuss et al. [2008] Numerical analysis of the impact of policy uncertainty on investment in abatement technology in a real options model via discrete time dynamic programming; a related study by the International Energy Agency is Yang et al. [2008]
- Empirical studies on impact of carbon taxes include Aghion et al. [2016] and Martinsson et al. [2022].
- There is also an empirical literature on climate policy uncertainty and climate policy uncertainty indices
- Optimal regulation: Aid and Biagini [2023] and many more

The model

- Consider a stylized electricity producer, who decides on *instantaneous production* $q_t \geq 0$ and on *investment* into abatement technology.
- Producer pays emission taxes represented by tax rate τ.
- She is a price taker (merit order system). Instantaneous profit:

$$\Pi(q, I, \tau, y) = p(y)q - C(q, I, \tau) + \nu_0(q)\tau \tag{1}$$

  Here y is some exogenous factor, $p(y)$ is the *price* and $C(q, I, \tau, y)$ the *cost function* for producing q units of electricity, given investment value I and tax rate τ. ν_0 models a tax rebate.
- C is increasing and convex in q, ν is increasing and concave.
- Producer chooses q_t to maximise instantaneous profit; optimal profit:

$$\Pi^*(I, \tau, y) = \max_{q \geq 0} \Pi(q, I, \tau, y). \tag{2}$$

- Often we consider the simpler case where q is fixed or where factor process is not there.

Investment in abatement technology

- Producer chooses rate $\boldsymbol{\gamma} = (\gamma_t)_{t\geq 0}$ at which she invests in abatement technology. For a given strategy $\boldsymbol{\gamma}$, the investment value I has dynamics

$$I_t = I_0 + \int_0^t \gamma_s \mathrm{d}s - \int_0^t \delta I_s \mathrm{d}s + \sigma W_t, \quad t \geq 0 \tag{3}$$

  where W is a Brownian motion, $0 \leq \delta < 1$ the depreciation rate and $\sigma \geq 0$ (typically small).
- We assume $\gamma_t \geq 0$ for all t (irreversible investment); $\mathcal{A}$ denotes the set of admissible strategies.
- Investment is subject to buildup- or transaction cost given by $\kappa\gamma^2$ (penalization of rapid build up of abatement technology).
- Investment is financed by borrowing at interest rate $r > 0$

Optimal investment problem

- Goal of the producer: choose strategy γ to maximize total profits up to time $T > 0$, that is

$$\max_{\gamma \in \mathcal{A}} \mathbb{E}_t \left[\int_t^T \left(\Pi^*(I_s, \tau_s, Y_s) - \gamma_s - \kappa \gamma_s^2 \right) e^{-r(s-t)} \mathrm{d}s + e^{-r(T-t)} h(I_T) \right] \tag{4}$$

- $h(\cdot)$ accounts for the residual value of the abatement technology at time T.
- We will solve this problem (numerically) via dynamic programming equation

Production function: filter technology

- Let X be the input (say, coal) with price $\bar{c}$ per unit.
- Amount of emission (CO_2) per unit of X is e_0. Filters $\Rightarrow$ emissions are reduced by $e_1(I)$.
- Total emission: $e(X, I) = X(e_0 - e_1(I))$, where abatement function $e_1(\cdot)$ is increasing, concave and bounded by e_0
- $Q(X)$ is electricity that can be produced from X units coal, for $Q(\cdot)$ increasing and concave.
- This gives the following cost function for producing q units of electricity

$$C(q, I, \tau) = Q^{-1}(q)(\bar{c} + \tau(e_0 - e_1(I))), \tag{5}$$

Example 2: Two technologies

- The energy producer has access to two production technologies, e.g. coal or gas and solar panels.
- Gas costs $c_b(y)$ per unit and emits e_b tons of CO_2 per unit.
- $Q_b(X)$ electricity produced with X units of gas.
- Green production has zero marginal cost, does not emit CO_2.
- $c_g I$ electricity produced green for given investment I.
- Operating cost for green technology $C_0(I)$

$$C(q, I, \tau) = \begin{cases} C_0(I) & \text{if } q - c_g I \leq 0, \\ C_0(I) + (c_b(y) + e_b \tau) Q_b^{-1}(q - c_g I) & \text{if } \ q - c_g I > 0, \end{cases} \quad (6)$$

Tax rate as finite state Markov chain

Tax process $(\tau_t)_{t\geq 0}$ is a finite state Markov chain with values $0 \leq \tau^1 < \cdots < \tau^K$ and switching intensities $g_{ij} = g_{ij}(y) \geq 0$

In the numerical experiments we consider examples with 2 states:

- Random tax increase. Here $\tau_0 = \tau^1$ but producer expects τ to increase to τ^2 at random later state, eg. as government implements international climate treaties
- Tax reversal. Here τ is initially in the high-tax state τ^2, but producer expects a correction (jump to τ^1 at a later date) perhaps due to a change in government ("Trump after Biden");

The tax scenarios

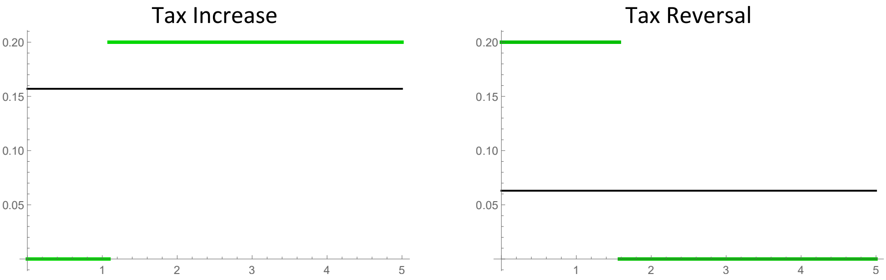

Figure: Tax policies. **black** deterministic tax rate, **green** random tax rate. In each panel the quantity $\mathbb{E}\left[\int_0^T \tau_s \mathrm{d}s\right]$ is identical for random and deterministic tax

Control problem and value function

Problem (4) is a stochastic control problem with value function $V^i(t,I,y) := V(t,I,\tau^i,y)$, $1 \le i \le K$. The associated HJB equation is a PDE system of the form

$$v_t^i + \Pi^*(I,\tau_i,y) - rv^i + \sum_{j=1}^{K}[v^j(t,I) - v^i(t,I]g_{ij}(t) + \sigma^2 v_{II}^i \tag{7}$$

$$+ \mathcal{L}^Y v^i + \sup_{0\le\gamma}\{v_I^i(\gamma - \delta I) - (\gamma + \kappa\gamma^2)\} = 0, \tag{8}$$

with the final condition $v^i(T,I,Y) = h(I)$. Here $\mathcal{L}^Y$ is the generator of the factor process Y (a diffusion)

Optimal strategy. Assume V is a classical solution. Then optimal investment rate is $\gamma^*(t,I,\tau,y) = (V_I(t,I,\tau,y) - 1)^+/2\kappa$ (Trade-off between expected future profits and current cost.)

Mathematical results

Assumptions

i. $\Pi^*(I, \tau, y)$ is continuous in (I, τ, y), increasing, and Lipschitz-continuous in I, y, uniformly in τ,

ii. $h(I)$ is increasing and Lipschitz.

Assumptions on Π^* cannot simply be imposed (unless if q is fixed) but can be verified under Lipschitz conditions on the data of the problem

Proposition. Under these assumptions, v is increasing, Lipschitz in I and y and Hölder in t and the unique viscosity solution of the HJB equation (7). Moreover, the optimal investment rate is bounded.

Proof is based on results from Pham [1998] and on comparison arguments

Mathematical analysis continued

- For $\sigma = 0$ we have examples for strict viscosity solutions
- If $\sigma > 0$ and if $\mathcal{L}^Y$ is strictly elliptic with sufficiently regular components we can show existence of a classical solution. Proof is based on a fixed point argument and on results for quasilinear parabolic equations from Ladyzenskaja et al. [1968].
- In general we need numerical techniques to solve the PDE system.
- We used the deep splitting method (an approximation method for semilinear P(I)DEs using backward induction and deep neural networks) studied eg. in Beck et al. [2021], Frey and Köck [2022], Germain et al. [2022].
- Method performs well, but numerical experiments time consuming

Numerical experiments: Setup and overview

- Throughout we consider the case where q is equal to $\bar{q} = 10$, $\delta = 0.05, \sigma = 0.05,\ T = 15$.
- Filter technology. Profit is increasing and concave in I, residual value $h(I) = 0$;
- Tax rate: 2 states $\tau^1 = 0$, $\tau^2 > 0$, transition intensity $g_{12} = 0.25, g_{21} = 0$ (random tax increase) resp. $g_{21} = g_{12} = 0.25$

We show results on

- Optimal investment rate for different buildup cost κ
- Comparison of average investment and emission reduction to a deterministic scenario with same average tax rate for tax reversal and random tax increase scenario

Optimal investment for tax increase scenario (filter)

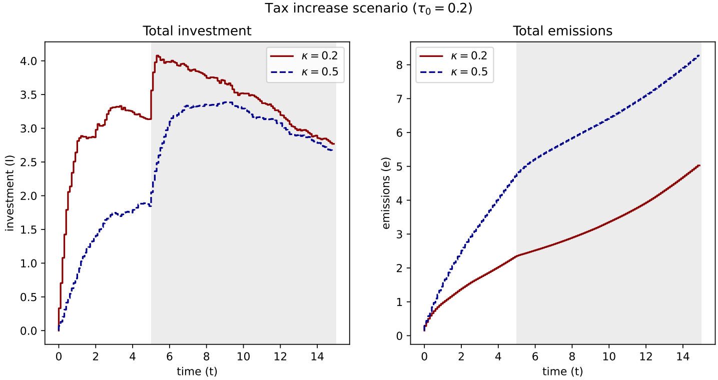

Figure: Optimal investment $I^*(t)$ for tax increase; left: random tax, right: constant tax. Note that there is a substantial amount of investment already before the jump in τ (hedging)

Optimal investment for tax reversal scenario (filter)

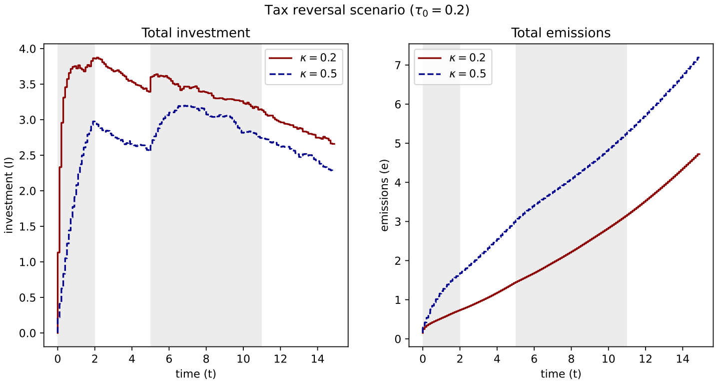

Figure: Optimal investment $I^*(t)$ for tax reversal; left: random tax, right: constant tax.

Average emissions (filter)

κ	random	constant
0.2	5.45	3.75
0.5	8.90	6.76

κ	random	constant
0.2	4.25	3.83
0.5	7.20	6.07

Table: left: random tax increase; right: tax reversal. The constant tax leads on average to lower emissions in both cases.

For the two technology case there is no clear ordering of the different tax policies.

Tax uncertainty and differential game

- Climate policy variables are the result of political processes. $\Rightarrow$ difficult to come up with 'correct' probabilistic model for tax dynamics, that is producer faces uncertainty (as opposed to risk).
- we therefore model optimal investment under tax uncertainty as stochastic differential game between *producer* and a malevolent opponent (*nature*).
- Producer chooses investment rate $\gamma \in \mathcal{A}$ and production $\boldsymbol{q}$ to maximize profits; nature chooses a worst case tax process $\boldsymbol{\tau}$ to minimize profits. $\Rightarrow$ Reward function

$$J(t, I, y, \boldsymbol{\tau}, \gamma, \boldsymbol{q}) = \mathbb{E}_t\Big[\int_t^T \big(\Pi(q_s, I_s, \tau_s, Y_s) - \gamma_s - \kappa\gamma_s^2 + \nu_0(q_s)\tau_s + \nu_1(\tau_s - \bar{\tau}(s))^2\big)e^{-r(s-t)}\mathrm{d}s + h(I_T)e^{-r(T-t)}\Big],$$

where $\nu_1(\cdot)$ penalizes deviation from anticipated tax plan.

The Bellmann-Isaacs equation

- Define $g(q,\tau;I,y) = \Pi(q,I,\tau,y) + \nu_0(q)\tau + \nu_1(\tau - \bar{\tau}(t))^2$
- We show that g admits a unique saddle point (q^*,τ^*) for every I,y. Denote by $G(I,y) = g(q^*(I,y),\tau^*(I,y),I,y)$ the corresponding saddle value. Then the Bellman Isaacs equation for the game reduces to the following standard HJB equation

$$u_t + G(I,y) + \mathcal{L}^Y u + \frac{\sigma^2}{2} u_{II} + \sup_{\gamma \geq 0} \left(\gamma u_I - \gamma - \kappa\gamma^2 \right) = ru\,. \qquad (9)$$

- If $\sigma^2 > 0$ (and some other regularity conditions) this equation has a unique classical solution.
- Equilibrium strategies are given by $q_t^* = q^*(I_t, Y_t), \tau_t^* = \tau^*(I_t, Y_t)$, $0 \leq t \leq T$ and $\gamma_t^* = (u_I(t, I_t, Y_t) - 1)^+ / 2\kappa$, $0 \leq t \leq T$

Special cases and qualitative properties of τ^*

- $\nu_0 \equiv 0 \Rightarrow \tau^*(q) > \bar{\tau}$, that is without rebate tax uncertainty leads to high expected taxes
- full abatement $(C_1 \equiv 0)$ and $\nu_0 > 0 \Rightarrow \tau^*(q) < \bar{\tau}$
- little abatement $(C_1 > \nu_0) \Rightarrow \tau^*(q) > \bar{\tau}$
- The anticipated produced amount q^* is lower then if taxes are equal to $\bar{\tau}$.

Summary and Conclusion

- For the filter technology random tax seems to be worse than deterministic benchmark;
- Results for the case with divisible investment (stochastic control) complement the real options approach of Fuss et al. [2008]. In particular, we see that there is *hedging* and buildup cost matter a lot.
- Further work
    - More simulations: differential game, two technologies, rebate etc.
    - Cost of capital: higher interest rate for borrowing than for investing
    - Equilibrium considerations (many small producers $\Rightarrow$ mean-field game)

P. Aghion, A. Dechezleprêtre, D. Hémous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1): 1–51, 2016. doi: 10.1086/684581.

R. Aid and S. Biagini. Optimal dynamic regulation of carbon emissions market. *Mathematical Finance*, 33(1):80–115, 2023.

Christian Beck, Sebastian Becker, Patrick Cheridito, Arnulf Jentzen, and Ariel Neufeld. Deep splitting method for parabolic PDEs. *SIAM Journal on Scientific Computing*, 43(5):A3135–A3154, 2021.

Rüdiger Frey and Verena Köck. Deep neural network algorithms for parabolic pides and applications in insurance and finance. *Computation*, 10, 2022. https://doi.org/10.3390/computation10110201.

S. Fuss, J. Szolgayova, and M. Obersteiner. Investment under market and climate policy uncertainty. *Applied Energy*, 85:708–721, 2008.

Maximilien Germain, Huyen Pham, and Xavier Warin. Approximation error analysis of some deep backward schemes for nonlinear pdes. *SIAM Journal of Scientific Computing*, 22:A28 –A56, 2022.

M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014.

O. A. Ladyzenskaja, V. A. Solonnikov, and N. N. Ural'ceva. *Linear and Quasilinear Equations of Parabolic Type*. American Mathematical Society, Providence, Rhode Island, 1968.

G. Martinsson, P. Stromberg, L. aszlo Sajtos, and C. Thomann. Carbon pricing and firm-level CO2 abatement: Evidence from a quarter of a century-long panel. working paper, European Corporate Governance Institute, 2022. Available at SSRN: https://ssrn.com/abstract=4206508 or http://dx.doi.org/10.2139/ssrn.4206508.

W. Nordhaus. Optimal greenhouse-gas reduction and tax policy in the 'DICE' model. *The American Economic Review*, 82:313–317, 1993.

Huyên Pham. Optimal stopping of controlled jump diffusion processes: a viscosity solution approach. *Journal of Mathematical Systems, Estimation and Control*, 8:1–27, 1998.

M. Yang, W. Blyth, and R. Bradley. Climate policy uncertainty and investment risk. Technical report, International Energy Association, Paris, 2008.