Contract theory and energy markets

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ETH Zürich

WPI-Workshop: Stochastics, Statistics, Machine Learning and their Applications to Sustainable Finance and Energy Markets, September 13, 2023

Outline

Motivation & intuition

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- The need for more flexibility in electric systems can be satisfied either
- ...by batteries or...
- ...a better use of demand flexibility potential.
- Possible to use distributed control of appliances.
- Also possible to use *demand–response programs*.

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- The number of days was limited at the inception.
- The period of High or Low price could be 3, 6, 9 or 12 hours.

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- DR programs reduce consumption level on average but at a significant infrastructure cost and with a significant variance in consumers' response.
- Need for better DR mechanisms!

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(i) α is the effort to reduce his consumption in mean;
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- The Principal (she) is a producer (or a retailer) subject to energy generation costs and to consumption volatility costs.

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$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T \Gamma_s \mathrm{d}\langle X \rangle_s + \frac{1}{2}R_A \int_0^T Z_s^2 \mathrm{d}\langle X \rangle_s,$$

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Results.

- Optimal contracting allows the system to bear more risk as the resulting volatility may increase;
- The control of the consumption volatility can lead to a significant increase of responsiveness.

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where

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Optimisation problem of the representative consumer:

$$V_0^A(\xi, \hat{\mu}) := \sup_{\mathbb{P} \in \mathcal{P}} \mathbb{E}^{\mathbb{P}}\left[U_A\left(\xi(X, \hat{\mu}) - \int_0^T \left(c(\nu_t^{\mathbb{P}}) - f(X_t)\right) \mathrm{d}t\right)\right], \tag{3}$$

where c is a cost function, f denotes the preference of the Agent toward his deviation consumption, and $U_A(x) = -\mathrm{e}^{-R_A x}$.

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- $\widehat{\mathbb{E}}^{\hat{\mu}}$ expectation under $\hat{\mu}$ (w.r.t the common noise).

Equivalence with contract on the common noise

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where $\overline{Z}_t^\mu := \widehat{\mathbb{E}}^{\hat{\mu}}\big[Z_t^\mu(\widehat{X}_t)\big]$.

▸ If the Principal can offer contract depending directly on the common noise, she can offer this contract, indexed by $\overline{\zeta}_t = \big(Z_t, \overline{Z}_t^\mu, \Gamma_t\big)$.

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▸ Contracting on $\hat{\mu}$ or W° leads in fact to the same form of contract.

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Her problem is reduced to a McKean–Vlasov control problem:

$$V^P := \sup_{\zeta\in\mathcal{V}} \mathbb{E}\Big[U^P\big(-\mathbb{E}^{\mu^L_T}[L_T]\big)\Big], \quad L_T = \xi_T + \int_0^T g(X_s)\mathrm{d}s + \frac{h}{2}\int_0^T \mathrm{d}\langle X\rangle_s,$$

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Two state variables: the conditional law of X (μ^X) and the conditional law of L (μ^L) $\Longrightarrow$ HJB techniques.

Optimal contract

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^{\star} + \frac{R_P}{R_A + R_P} \overline{u}^P_{\mu^X},$$

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leads to the optimal contract:

$\xi_T := \xi_0$

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$$\underbrace{+\frac{1}{2}R_A \int_0^T \bigg((Z^\star_s)^2 \Big(\mathrm{d}\langle X\rangle_s - (\sigma^\circ)^2 \mathrm{d}s\Big) + \frac{R_P^2}{(R_A + R_P)^2}(\sigma^\circ)^2 \big(\overline{u}^P_{\mu^X}\big)^2 \mathrm{d}s \bigg)}_{\text{Compensation for risk due to the risk aversion of the consumer } (R_A)}.$$

Gain in efforts

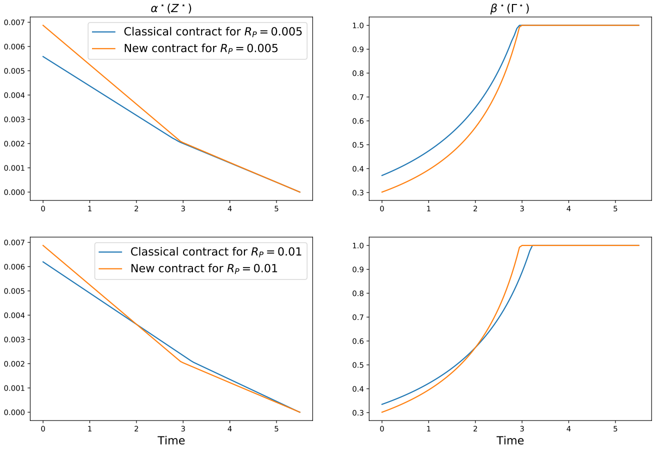


Figure: Effort on the drift (left) and on the volatility (right). Variation with respect to R_P: $R_P = 0.005$ (up) and $R_P = 0.01$ (bottom).

Gain in efforts (2)

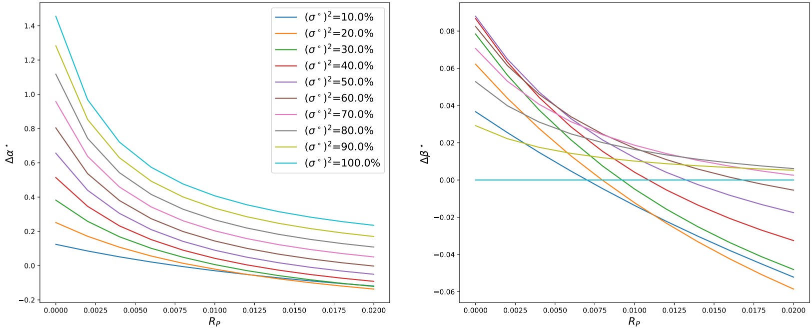


Figure: Relative gain on efforts. Variation with respect to R_P and σ°.

Gain in utility for the Principal

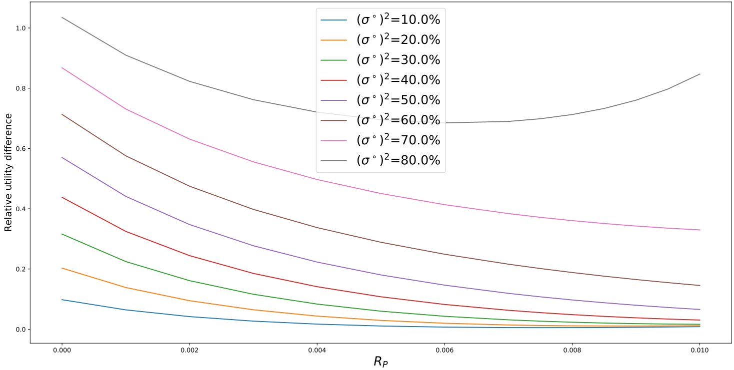


Figure: Relative utility difference. Variation with respect to R_P and σ°.

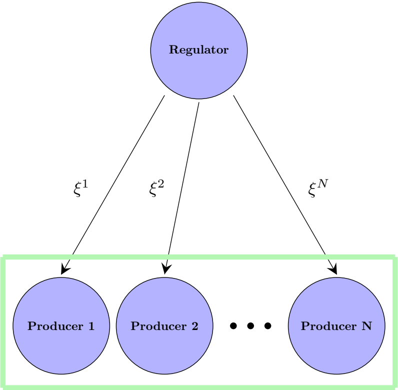
Outline

1. An application in demand–response management
   - Motivation & intuition
   - A moral hazard approach
   - A Stackelberg game with many consumers
   - Optimal contract
   - Principal's Problem
2. A model for pollution regulation
   - Setting and results
   - Numerical results

A model for pollution regulation

A **regulator** tries to provide a system of electricity **producers** with incentives to

- use clean energies;
- invest to reduce the emission levels of other means of production.



A model for pollution regulation

- We place ourselves after the auction phase where **producers** have bid their cost functions.
- We are given a network structure where nodes are **producers**, with local demand D, production capacity Q, with power flows and transmission lines with losses r
- The role of the **regulator** is to decide the production levels q and energy flows ϕ^e at each node...

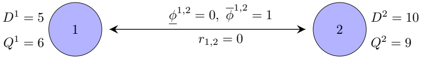


Figure: Example of the network constraints.

- ...and to choose a bonus/malus system to incentivise pollution reduction, through terminal payments ξ

The setting

Each **producer** has an impact on the **drift** of the overall pollution L, thanks to their **efforts**.

$$L_t = L_0 + \sum_{i=1}^{N} \int_0^t (1 - a_s^i) p_i(q_s^i) \mathrm{d}s + \sigma W_t^a.$$

The **regulator** tries to minimise the social cost

$$\inf_{(q,\phi,\xi)\in\mathcal{C}} \mathbb{E}^{\mathbb{P}^{a^\star(q,\xi)}} \bigg[\sum_{i=1}^{N} \int_0^T c_i(q_s^i) \mathrm{d}s + \int_0^T \Lambda(L_s - \ell_o) \mathrm{d}s + \sum_{i=1}^{N} \xi^i \bigg],$$

where $a^\star(q,\xi)$ is a **Nash equilibrium** for the contracts (q,ϕ,ξ).

The best-reaction function of each **producer**, given actions chosen by the others

$$V_0^i(a^{-i},\xi^i,q) := \sup_{a^i\in\mathcal{A}_i} \mathbb{E}^{\mathbb{P}^a} \bigg[U_A \bigg(\xi^i - \int_0^T \big(h_i(a_s^i) + c_i(q_s^i) \big) \mathrm{d}s \bigg) \bigg].$$

The setting

- Each **producer** has different technologies: he produces at the lowest marginal cost until the cheapest available technology is saturated.
- The network structure does not impact the resolution method, and only a finite dimensional optimisation problem.

Main results

• The set of admissible remunerations can be represented as the terminal values of the family of processes

$$Y_t^{y,q,Z} = y + \int_0^t f(q_s, Z_s)\mathrm{d}s + \int_0^t Z_s \mathrm{d}L_s,\ t \in [0, T],$$

with the function $f : \mathbb{R}_+^N \times \mathbb{R}^N \longrightarrow \mathbb{R}^N$ given by

$$f^i(q,z) := h_i\big(a^{\star,i}(q,z)\big) + c_i(q^i) - z^i \sum_{j=1}^N \big(1 - a^{\star,j}(q,z)\big)p_j(q^j) + \frac{\rho\sigma^2}{2}(z^i)^2,$$

and

$$a^{\star,i}(q,z) := \mathrm{argmin}_{a\in A_i}\big\{h_i(a) - z^i(1-a)p_i(q^i)\big\}.$$

Main results

• The value of the principal and the optimal contract are characterised by the HJB equation

$$-v_t - G(t,\ell,v_\ell,v_{\ell\ell}) = 0,\ (t,\ell)\in[0,T)\times\mathbb{R},\ v(T,\ell)=0,\ \ell\in\mathbb{R},$$

with $G:[0,T]\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}\longrightarrow\mathbb{R}$ given by

$$G(t,\ell,\alpha,\gamma) := \inf_{(z,q,\phi)\in\mathbb{R}^N\times\hat{P}_t}\{g(\alpha,z,q,\phi)\} + \frac{1}{2}\gamma\sigma^2 + \Lambda(\ell-\ell_0),$$

and $g:\mathbb{R}\times\mathbb{R}^N\times\hat{P}\longrightarrow\mathbb{R}$ defined by

$$g(\alpha,z,q,\phi) := \sum_{i=1}^N\bigg(\alpha\big(1-a^{\star,i}(q,z)\big)p_i(q^i) + h_i\big(a^{\star,i}(q,z)\big) + \frac{\rho\sigma^2}{2}(z^i)^2 + 2c_i(q^i)\bigg).$$

Main results

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• This PDE has a unique viscosity solution with polynomial growth, which turns out to be C^1 $\longrightarrow$ enough to prove existence of an optimal contract

$$\xi^\star := y + \int_0^T f(q_s^\star, Z_s^\star)\mathrm{d}s + \int_0^T Z_s^\star \mathrm{d}L_s$$

Calibration

Inspired by the Chilean electricity market, we consider a network with three nodes representing the North, South and Centre regions of the country.

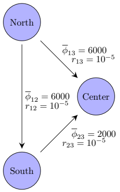


Figure: Characteristics of the network.

Demands	Value	Capacities	Value	Parameter	Value
D^1	3000 MW	Q^1	6000 MW	σ	200 tons of CO_2 per square root of time
D^2	1000 MW	Q^2	2000 MW	T	90 days
D^3	6000 MW	Q^3	12000 MW	$\mathrm{d}t$	1 hour

Table: Values of the parameters in the numerical simulations.

Calibration

• There are three technologies to produce power, namely coal, gas and solar. Each technology is assumed to be identical for each node.

Technology	Cost (dollars per MWh)	Pollution (tons of CO_2 per MWh)
solar	0	0
coal	40	1
gas	80	0.5

Table: Cost and pollution of the technologies.

• Cost functions are piece-wise linear for each producer.

Without regulation

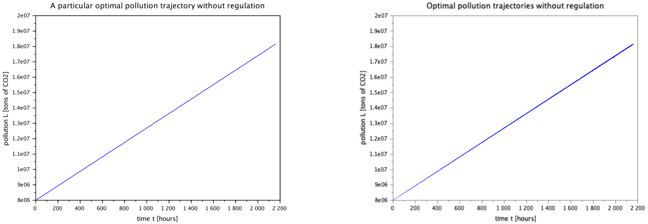


Figure: Pollution without regulation (30,000 simulations).

With regulation

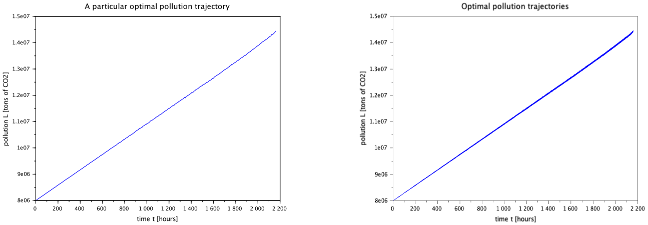


Figure: Pollution trajectories under regulation (30,000 simulations).

Contracts reduce the increment in the pollution levels by more than 30% percent, from 10,000,000 tons of CO_2 without regulation, to around 6,500,000 tons of CO_2.

With regulation

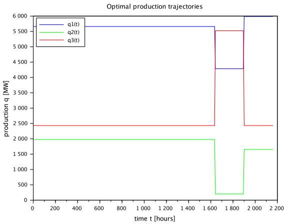
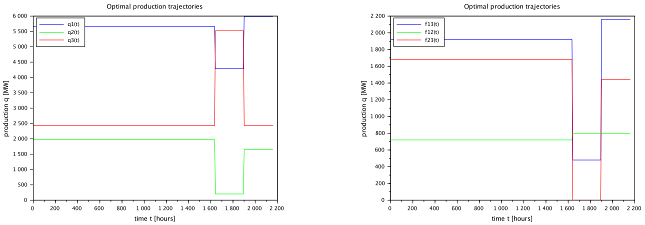


Figure: Production and transmission plans under regulation (30,000 simulations).

• First, Centre node produces exactly 2400 MW, with solar technology and no pollution. Leftover is provided by the others. Node in the North produces 5600 MW, while the South produces 2000 MW and receives a flow of 700 MW from the North.

With regulation



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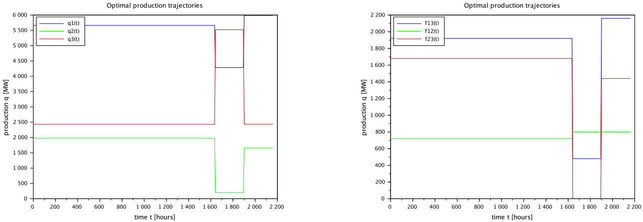
• Second, node in the Centre cover most demand and produces 5500 MW. The node in the North contributes 500 MW and no flow is received from South. This occurs when the increase of pollution has been controlled for enough time and it becomes less costly to allow the node in the Centre to produce power.

With regulation



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- Second, node in the Centre cover most demand and produces 5500 MW. The node in the North contributes 500 MW and no flow is received from South. This occurs when the increase of pollution has been controlled for enough time and it becomes less costly to allow the node in the Centre to produce power.
- Third, we get back to a situation similar to the first period.

Thank you for your attention